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USE OF THE FOURIER ANALYZER FOR FAR-FIELD SPECKLE ANALYSIS.(U)
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USE OF THE FOURIER ANALYZER FOR FAR-FIELD SPECKLE ANALYSIS

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4 August 1976

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A sequel to Surface Information From Far-Field Speckle Analysis (Report No. RR-TR-76-2) is presented. A clarification of how the theoretical results of that report relate to experimental data input for the HP 5451B Fourier analyzer is undertaken. The relationship between time, frequency, and position as well as the appropriate choice of data intervals are discussed. The rectangular window width is shown to be incorporated into a function defined in the previous work which determines resolution of surface detail.

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CONTENTS

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Helpful discussions with R. R. Lattanzi about programming and interpreting results of the HP 5451B Fourier analyzer are appreciated.

I. INTRODUCTION

The Fourier transform of a function I(t) is given by

$$S(f) = \int_{-\infty}^{\infty} I(t) \exp j(-2\pi ft)\, dt \quad . \tag{1}$$

S(f) will have δ-function values if I(t) is sinusoidal. Except for the finite amplitude, an equivalent expression can be obtained by integration over only one period of I(t), provided only the values of f for which f = m/T (m = integer) are used:

$$S(m \triangle f) = \int_{0}^{T} I(n \triangle t) \exp j(-2\pi m \triangle f\ t)\, dt \quad , \tag{2}$$

where $\triangle f = 1/T$. Equation (2) is the basis of the discrete Fourier transform (DFT). The HP 5451B Fourier analyzer evaluates Equation (2) with the integration replaced by a summation:

$$S'(m \triangle f) = \triangle t \sum_{n=0}^{N-1} I(n \triangle t) \exp j(-2\pi m \triangle f\ n \triangle t) \quad . \tag{3}$$

In the last equation, the period is divided into equal segments $\triangle t = T/N$. Therefore, $\triangle t$ must be sufficiently small so that it will resolve pertinent structure (i.e., N must be large enough). Only for this condition will a meaningful frequency spectrum be displayed. Also, T must be sufficiently large so that the frequency resolution $\triangle f = 1/T$ is adequate.

The Fourier analyzer output F is actually 1/T times $S'(m \triangle f)$. This insures that if an input amplitude at frequency n is I_0, then F has an amplitude output of $I_0/2$.

When analyzing a segment of a periodic waveform, care must be taken to insure that the "window" is observed over a complete period or integral number of periods. If this is not done, error will result. In the case of far-field speckle data, there is no exact periodicity. The theory developed in Report No. RR-TR-76-2 takes the error induced by integration over a finite window into consideration. The Fourier analyzer can be used directly without concern about the exact value for the window. More discussion on this matter is taken up later.

In Report No. RR-TR-76-2, the following transform was defined:

$$F'_{-x_1,x_1}(u) = \int_{-x_1}^{x_1} I'(x)\, [\exp j(-ux)]\, dx \quad , \tag{4}$$

where $x = \sin\theta$ (θ = angle of detection with respect to object surface normal). For $I'(x) = I'(-x)$ the relation between the real part of F'_{-x_1,x_1} and the real part of the transform with integration limits from 0 to x_1 is

$$\mathrm{Re}\left\{F'_{-x_1,x_1}(u)\right\} = 2 \quad \mathrm{Re}\left\{F'_{0,x_1}(u)\right\} \quad . \tag{5}$$

The DFT obtained by the Fourier analyzer is

$$F(m\,\Delta f) = \frac{\Delta t}{T} \sum_{n=0}^{N-1} I(n\,\Delta t)\ \exp\ j(-2\pi m\,\Delta f\ n\,\Delta t) \quad . \tag{6}$$

The desired DFT is

$$2F'_{0,N-1}(m\,\Delta u) = 2\,\Delta x \sum_{n=0}^{N-1} I'(n\,\Delta x)\ \exp\ j(-m\,\Delta u\ n\,\Delta x) \quad . \tag{7}$$

Equivalence between Equations (6) and (7) is obtained by letting

$$\left.\begin{aligned} \Delta x &= \omega_0\,\Delta t \\ \Delta u &= \frac{2\pi}{\omega_0}\,\Delta f \end{aligned}\right\} \quad . \tag{8}$$

Equation (7) becomes

$$2F'_{0,N-1}(m\,\Delta u) = 2\omega_0\Delta t \sum_{n=0}^{N-1} I'(n\omega_0\Delta t)\ \exp\ j(-2\pi m\,\Delta f\ n\,\Delta t) \quad . \tag{9}$$

Defining $I'(\omega_0 n\,\Delta t) = I(n\,\Delta t)$ yields

$$2F'_{0,N-1}(m\,\Delta u) = 2\omega_0 T\ F(m\,\Delta f) \quad ; \tag{10}$$

hence,

$$\mathrm{Re}\left\{F'_{-x_1,x_1}(u)\right\} = 2\omega_0 T \quad \mathrm{Re}\left\{F(m\,\Delta f)\right\} \quad . \tag{11}$$

The relation between u and the variable representing displacement $\triangle$ on the illuminated surface is $2k\,\triangle = u$. It is noted that u is dimensionless. From Equation (8) it is seen that the real part of the transform F' as defined in Report No. RR-TR-76-2 (essentially an autocorrelation integral within a resolution length δ_p) is equal to $2\omega_0 T$ times the real part of the DFT obtained by the Fourier analyzer with time and frequency related to $x = \sin\theta$ and $u = 2k\,\triangle$ by Equation (8).

It should be noted that $\triangle x = \omega_0 \triangle t$ requires a variable angular scan rate. Because $\triangle x = \cos\theta\ \triangle\theta$, then $d\theta/dt = \triangle x\ (\cos\theta\ \triangle t) = \omega_0/\cos\theta$. Hence, the angular scan rate goes as the reciprocal of $\cos\theta$.

II. RELATION BETWEEN p(b) AND THE RECTANGULAR WINDOW

In Report No. RR-TR-76-2, a function

$$F'(u) = \int_{-x_1}^{x_1} A^*A[\exp j(-ux)]\ dx \tag{12}$$

was defined, where $A^*A = I(x)$ as presented in Equation (4) and $x = \sin\theta$ (θ is arc angle). This could be written (setting $u = 2\pi u'$)

$$F'(2\pi u') = \int_{-\infty}^{\infty} A^*A\ W(x)\ [\exp j(-2\pi u'x)]\ dx \quad , \tag{13}$$

where

$$W(x) = \begin{cases} 1 \ , & -x_1 \le x \le x_1 \\ 0 \ , & \text{Otherwise} \end{cases} \quad . \tag{14}$$

W(x) is the window function. It is seen then that F' is the Fourier transform of the rectangular window function times the field intensity A*A.

During the derivation of explicit forms for F', a function p(b/2k) where $b = 2k(\xi - \alpha + \pi u'/k)$ was introduced. This function was not purely a window function, but incorporates it because it was obtained by integrating over the finite limits shown in Equation (12). The relation between p and W is given as

$$p\left(\frac{b}{2k}\right) = \mathcal{F}\left\{W(x)\ [\exp -j2k(\xi - \alpha)x]\ (1 - x^2)\right\} \quad , \tag{15}$$

where $\mathcal{F}$ denotes a Fourier transform. It is seen that p also incorporates a phase obtained from factoring out part of the product $A(\xi,\eta)A^*(\alpha,\beta)$; furthermore, the factor $(1 - x^2) \equiv \cos^2 \theta$ is incorporated.

For the case of x_1 very small and $\xi = \alpha$, p(b/2k) should become essentially the transform of the rectangular window which is a sinc function. It is recalled that

$$p(b/2k) = \frac{2}{b}\left\{-\frac{2x_1}{b}\cos bx_1 + \left(1 - x_1^2 + \frac{2}{b^2}\right)\sin bx_1\right\} \quad . \qquad (16)$$

The significance of $\xi = \alpha$ will be taken up shortly. When $x_1 \to 0$ and b is small, Equation (16) becomes

$$p\left(\frac{b}{2k}\right) \to \frac{2}{b}\left\{-\frac{2x_1}{b} + bx_1 + \frac{2}{b^2}\,bx_1\right\} = 2x_1 \quad .$$

This is the same as the limit of $2x_1 \sin bx_1/(bx_1)$ ($2x_1$ times a sinc function) for sufficiently small bx_1. Now it is assumed $x_1 \to 0$ and $b \geq 10$. Equation (16) becomes

$$p(b/2k) \to \frac{2}{b}\left\{\frac{2x_1}{b}\cos bx + \sin bx_1\right\} \quad .$$

Because x_1/b is so small, this further goes to

$$p(b/2k) \to \frac{2\sin bx_1}{b} = 2x_1\left(\frac{\sin bx_1}{bx_1}\right) \quad .$$

Again this is $2x_1$ times a sinc function.

The significance of $\xi = \alpha$ can be visualized when the distant source is a point. For this case

$$A(p_1) = a_0\,\delta(\xi - \xi_0)\,\delta(\eta - \eta_0) \quad . \qquad (17)$$

Now $A(\theta)$ is given by

$$A(\theta) = \frac{1}{j\lambda R}\int_S A(p_1)\,(\exp jkr)\cos\theta\,ds$$

or

$$A(\theta) = \frac{\cos\theta}{j\lambda R} \exp jkR \int_{\xi,\eta} a_0\, \delta(\xi - \xi_0)\, \delta(\eta - \eta_0)$$

$$\times \exp(-jk\xi \sin\theta)\, d\xi\, d\eta \quad .$$

Therefore, A*A is given by

$$A^*A(\theta) = \frac{\cos^2\theta}{(\lambda R)^2} \int_{\xi,\eta}\int_{\alpha,\beta} a_0^2\, \delta(\xi - \xi_0)\, \delta(\eta - \eta_0)\, \delta(\alpha - \xi_0)\, \delta(\beta - \eta_0)$$

$$\times \exp[-jk(\xi - \alpha)\sin\theta]\, d\xi\, d\eta\, d\alpha\, d\beta \quad . \tag{18}$$

The δ-functions force the following result:

$$A^*A(\theta) = \frac{a_0^2 \cos\theta}{(\lambda R)^2} \quad . \tag{19}$$

Equation (19) was obtained because the integral gives no contribution for $\xi \neq \alpha$. Hence the exponential term in Equation (18) vanishes. Because p incorporated this factor, it can be neglected in Equation (15). For $\sin\theta = x$ small, $p(b/2k) \rightarrow \mathcal{F}\{W(x)\}$. Thus, for a point source and observation window perpendicular to the distance to the source (as well as small) the only significant values for $p(b/2k)$ become the Fourier transform of the rectangular window function. As a matter of fact, F' itself becomes the same, except for a factor:

$$F' = \int_{-x_1}^{x_1} \frac{a_0^2}{(\lambda R)^2} (\exp -j2\pi u'x)\, dx$$

or

$$F' = \frac{a_0^2}{(\lambda R)^2} \int_{-\infty}^{\infty} W(x)\, (\exp -j2\pi u'x)\, dx$$

or

$$F'(2\pi u') = \frac{a_0^2}{(\lambda R)^2} \frac{\sin 2\pi u'x_1}{2\pi u'x_1} \text{ ; point source and } x_1 \text{ small} \quad . \tag{20}$$

It may be of some question as to what effect the small side oscillations of $p(b/2k)$ may have on F' in general. The effect should be to decrease the value of the correlation peak by a small factor and decrease the shoulders of the correlation curve by a smaller factor. This amounts to only a small loss of resolution in the correlation value obtained. Oscillations will not appear explicitly because they are integrated over.

III. CONCLUSIONS

Applicability of using the Fourier analyzer in far-field speckle measurements to obtain discrete Fourier transforms which are essentially autocorrelation functions with a resolution δ_p [dependent upon the function $p(b/2k)$] has been confirmed. The manner in which the function p is related to the transform window has been clarified. The function p changes with increasing window size to give better resolution in the desired autocorrelation lengths. The position variable $x = \sin\theta$ which is related to the time variable that is read into the Fourier analyzer must change at constant rate. This implies a variable angular scan rate.

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